CH-2

Equation

"Mathematical Statement of Equality"

$x \rightarrow$ variable

$y = 2x + 4$

$y \downarrow$ dependent Variable

$x \rightarrow$ Independent

Polynomial :

degree

$ax^{n} + bx^{n-1} + cx^{n-2} + dx^{n-3} + \ldots$

coefficient, variable, constant

If $n=1$ $\quad ax + b = 0 \longrightarrow$ Linear equation

If $n=2$ $\quad ax^2 + bx + c = 0 \longrightarrow$ Quadratic equation

If $n=3$ $\quad ax^3 + bx^2 + cx + d = 0 \longrightarrow$ Cubic equation

$\boxed{\text{degree} = \text{No. of Solution}}$

Highest power coefficient will never be equal to zero.

$a \neq 0$

★ Linear eq. is of straight line.

→ Linear equation in one variable :-

$$7x - 28 = 0$$
$$7x = 28$$
$$x = \frac{28}{7}$$
$$\boxed{x = 4}$$

→ Linear equation in two variables :-

SUBSTITUTION METHOD

$$x + y = 20$$
$$x - y = 4$$

$$x = 20 - y$$

Putting in (ii)

$$20 - y - y = 4$$
$$-2y = -16$$
$$\boxed{y = 8}$$

Now, $x = 20 - y$

$$x = 20 - 8$$
$$\boxed{x = 12}$$

ELEMINATION METHOD

$$x + y = 20$$
$$x - y = 4$$

$$x + \cancel{y} = 20$$
$$+ \quad x - \cancel{y} = 4$$
$$2x = 24$$
$$\boxed{x = 12}$$

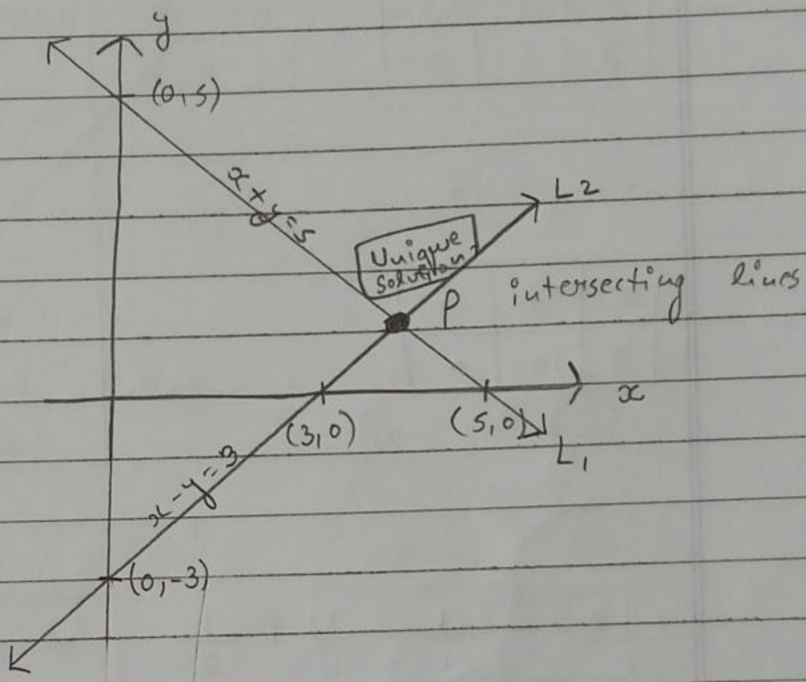
$$\cancel{x} + y = 20$$
$$- \quad \cancel{x} - y = 4$$
$$- \quad + \quad -$$
$$2y = 16$$
$$\boxed{y = 8}$$

★ $a_1x + b_1y = c_1$
$a_2x + b_2y = c_2$

If $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ then equation have uniques solution

(i) $x + y = 5$

x	0	5
y	5	0



$x - y = 3$

x	0	3
y	-3	0

$\frac{1}{1} \neq \frac{1}{-1}$ Unique solution of this eq.

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ Intersecting Lines, Unique Solution

Ex. $2x + ky = 70$
$3x + 8y = 80$

If both the equation has unique solution then find K.

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

$\frac{2}{3} \neq \frac{K}{8}$

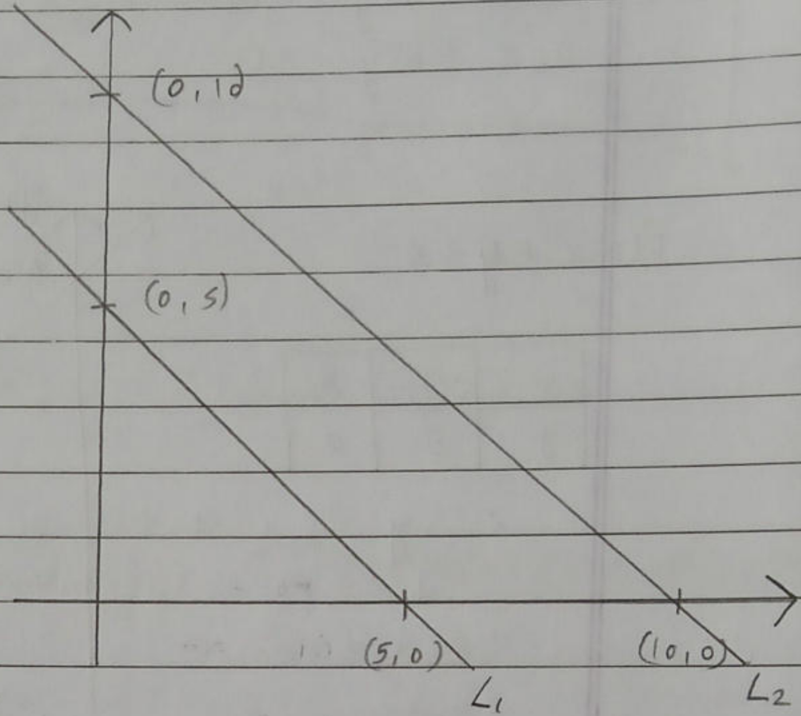
$\boxed{K \neq \frac{16}{3}}$ So, K should not be equal to $16/3$.

(ii) $x + y = 5$

x	0	5
y	5	0

$x + y = 10$

x	0	10
y	10	0



$\frac{1}{1} = \frac{1}{1} \neq \frac{1}{2}$ Parallel Lines

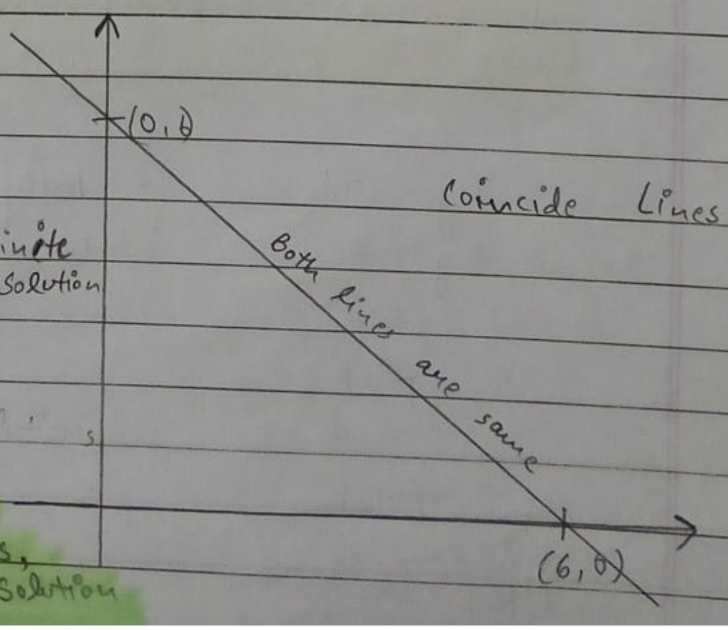
No Solution

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ Parallel Lines, No solution

(iii) $x + y = 6$

$3x + 3y = 18$

$\frac{1}{3} = \frac{1}{3} = \frac{1}{3}$ Infinite Solution



$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Coincide Lines, Infinite No. of Solution

Quadratic Equation

degree - 2

$$ax^2 + bx + c = 0 \quad , \quad a \neq 0$$

Spilting Method

$x^2 - 5x + 6 = 0$

$x^2 - 2x - 3x + 6 = 0$

$x(x-2) - 3(x-2) = 0$

$(x-2)(x-3) = 0$

$x - 2 = 0 \quad | \quad x - 3 = 0$

$\boxed{x = 2} \quad | \quad \boxed{x = 3}$

$x^2 - 5x - 6 = 0$

$x^2 - 6x + x - 6 = 0$

$x(x-6) + 1(x-6) = 0$

$(x-6)(x+1) = 0$

$x - 6 = 0 \quad | \quad x + 1 = 0$

$\boxed{x = 6} \quad | \quad \boxed{x = -1}$

Formula Method

$$\alpha, \beta = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Types of roots / zeroes

$$\frac{-b + \sqrt{D}}{2a} \qquad \frac{-b - \sqrt{D}}{2a}$$

$$\boxed{D = b^2 - 4ac}$$

- $D = 0$ two equal roots
- $D > 0$ two Distinct root
- D is a perfect square.
  two Rational and Distinct roots.
- D is not a perfect square (+ve)
  two irrational and Distinct roots.

□ $D < 0$ Imaginary roots

$\sqrt{-4} = \sqrt{-1 \times 4} = 2\sqrt{-1}$

$= 2(i)$

imaginary Number

★ Square of a negative number is imaginary Number

□ Sum of Roots & product of roots.

$\frac{-b + \sqrt{b^2 - 4ac}}{2a}$ $\quad$ $\frac{-b - \sqrt{b^2 - 4ac}}{2a}$

$\boxed{\alpha + \beta = \frac{-b}{a}}$ $\quad$ $\boxed{\alpha\beta = \frac{c}{a}}$

★ Formation of equation

$x^2 -$ (sum of roots) x + product of roots $= 0$

Ex. $(2, 5)$

$x^2 - 7x + 10 = 0$

Identities

$$(\alpha+\beta)^2 = \alpha^2+\beta^2+2\alpha\beta$$
$$(\alpha-\beta)^2 = \alpha^2+\beta^2-2\alpha\beta$$
$$(\alpha-\beta)^2 = (\alpha+\beta)^2-4\alpha\beta$$
$$(\alpha+\beta)^3 = \alpha^3+\beta^3+3\alpha\beta(\alpha+\beta)$$
$$(\alpha-\beta)^3 = \alpha^3-\beta^3-3\alpha\beta(\alpha-\beta)$$

$$\alpha^2+\beta^2 = (\alpha+\beta)^2-2\alpha\beta$$
$$\alpha-\beta = \sqrt{(\alpha+\beta)^2-4\alpha\beta}$$
$$\alpha^2-\beta^2 = (\alpha+\beta)(\alpha-\beta)$$
$$\alpha^3+\beta^3 = (\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)$$
$$\alpha^3-\beta^3 = (\alpha-\beta)^3+3\alpha\beta(\alpha-\beta)$$

→ Cubic Equation

$$ax^3+bx^2+cx+d=0$$

degree = 3
Value of x = 3

$$x^3-6x^2+11x-6=0$$

A. $(-1,1,-2)$ ✓B. $(1,2,3)$ C. $(-2,2,3)$ D. $(0,4,-5)$

Sum of Roots $= \frac{-b}{a} = \frac{-(-6)}{1} = 6$

Product $= \frac{-d}{a}$

Irrational roots are consecutive

$a+\sqrt{b}$, $a-\sqrt{b}$

$2+\sqrt{3}$, $2-\sqrt{3}$ $\rightarrow$ $x^2 - (\text{sum})x + (\text{product})$

$x^2 - 4x + 1 = 0$

Sum $= 2+\cancel{\sqrt{3}} + 2 - \cancel{\sqrt{3}} = 4$

Product $= 2+\sqrt{3} \times 2-\sqrt{3}$

$\boxed{\text{Condition}}$: If one root is reciprocal of the other root then $a=c$

* Only applicable in case of infinity

If $\sqrt{6+\sqrt{6+\sqrt{6+\infty}}}$

2 3 — 3 is the ans.

$\sqrt{6-\sqrt{6-\sqrt{6-\infty}}}$

2 3 — 2 is the ans.

$\sqrt{6\sqrt{6\sqrt{6\infty}}}$

No. sign the ans. is 6